ENS Paris-Saclay

The Logic of Bunched Implications Is Undecidable

Galatos, Jipsen, Knudstorp, Ramanayake

Presented by Anusha Gupta

ENS Paris Saclay

Département d'informatique

October 2026

Why care about BI?

Why care about BI?

RESOURCES!

Why care about BI?

RESOURCES!

Additive reasoning

Ordinary intuitionistic connectives: $\wedge$, $\rightarrow$.

Assumptions may be reused or discarded.

It allows **weakening** and **contraction**.

$$\frac{\Gamma \vdash B}{\Gamma, A \vdash B}$$

Resource-sensitive reasoning

Multiplicative connectives : $\star$, $-\!\!*$.

Resources cannot generally be duplicated or discarded

Does not allow **weakening** and **contraction**

$$\frac{\Gamma, A, A \vdash B}{\Gamma, A \vdash B}$$

→ BI is important as it is used to reason about programs that manipulate memory. (Here memory is a resource!)

→ BI is important as it is used to reason about programs that manipulate memory. (Here memory is a resource!)

→ Despite its elegant proof calculus, BI’s decidability remained a prominent open question.

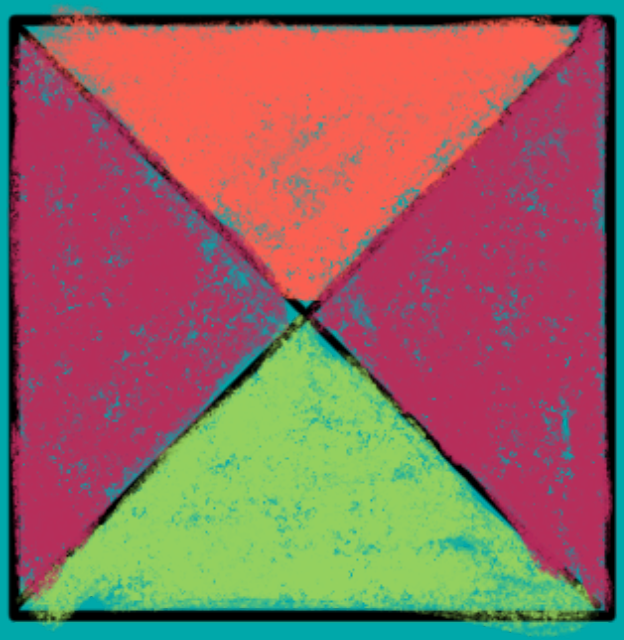
→ BI is important as it is used to reason about programs that manipulate memory. (Here memory is a resource!)

→ Despite its elegant proof calculus, BI’s decidability remained a prominent open question.

→ Earlier papers had even claimed decidability, and this work resolves the question

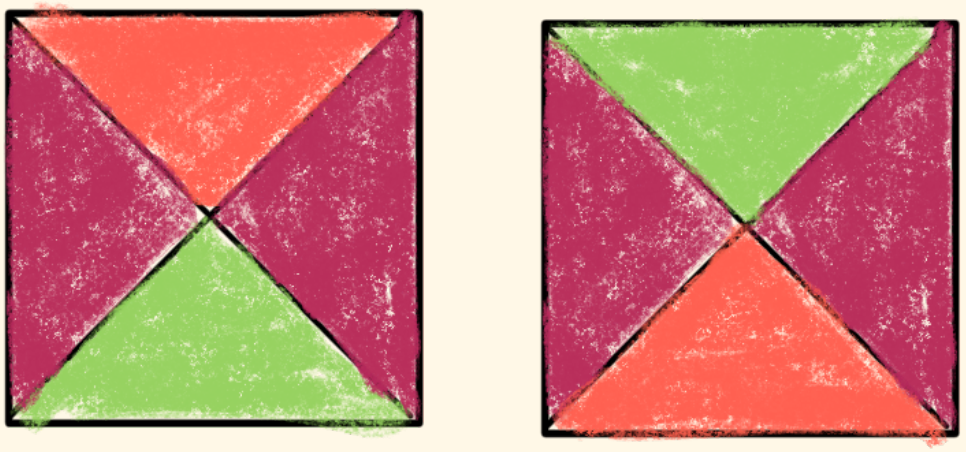
And what is a Wang tiling?

And what is a Wang tiling?





Wang Tiling



A square with a colour on each of its four edges.

Wang Tiling





The Tiling Problem

A square with a colour on each of its four edges.

Wang Tiling





A square with a colour on each of its four edges.

The Tiling Problem

Adjacent edges must match:

Wang Tiling





A square with a colour on each of its four edges.

The Tiling Problem

Adjacent edges must match:

→ Horizontally: right = left.

→ Vertically: bottom = top.

Wang Tiling





A square with a colour on each of its four edges.

The Tiling Problem

Adjacent edges must match:

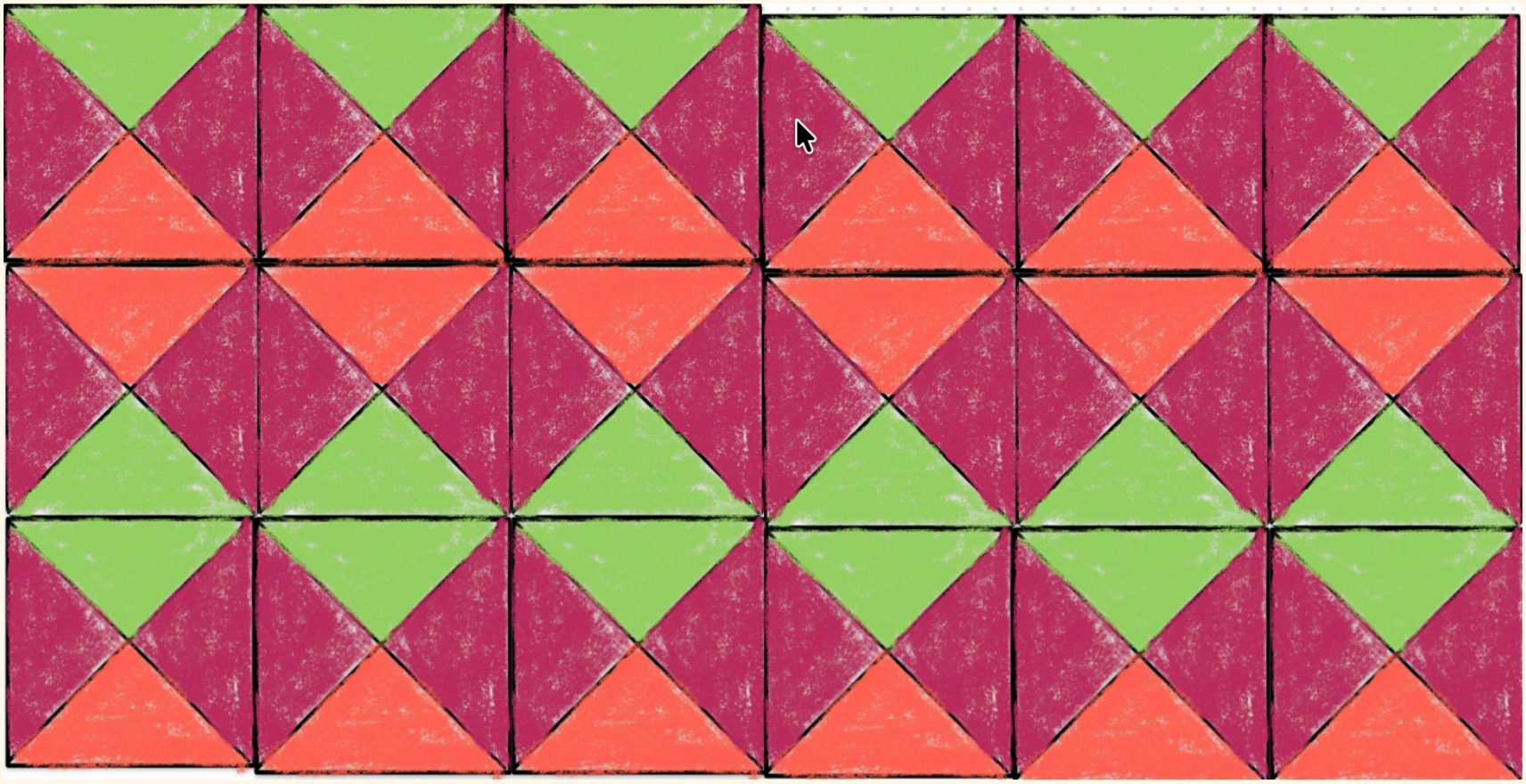
→ Horizontally: right = left.

→ Vertically: bottom = top.

Input: a finite set of tiles W.

Question: can W tile the entire infinite grid $\mathbb{N}^2$?

An example



what we know?

The Wang tiling problem is undecidable!!!

what we know?

The Wang tiling problem is undecidable!!!

well that’s a bummer…

The Reduction

from Wang tiling problem to BI’s provability problem

October 2026

The reduction

W set of tiles $\rightsquigarrow \varphi_W$

The reduction

$$W \text{ set of tiles} \rightsquigarrow \varphi_W$$

$$W \text{ tiles } \mathbb{N}^2 \iff \mathrm{BI} \not\models \varphi_W$$

$$W \text{ tiles } \mathbb{N}^2 \iff \mathrm{BI} \not\models \varphi_W$$

Forward direction:

If W tiles the grid, then we have a counter example our BI formula.

$$W \text{ tiles } \mathbb{N}^2 \iff \mathrm{BI} \not\models \varphi_W$$

Forward direction:

If W tiles the grid, then we have a counter example our BI formula.

Backward direction:

If the formula has a counterexample then we have a way of tiling the grid

significance

significance

it settles a long-standing open question: BI is undecidable, despite earlier claims that it was decidable.

significance

it settles a long-standing open question: BI is undecidable, despite earlier claims that it was decidable.

the authors identify gaps in those earlier proofs

significance

it settles a long-standing open question: BI is undecidable, despite earlier claims that it was decidable.

the authors identify gaps in those earlier proofs

the result extends beyond BI to a broader family of related logics, including GBI

Thank you! … any questions?